Fiber Properties

Introduction

- optical attenuation
- fiber dispersion,
- the effects of fiber nonlinearities.

Fiber Losses

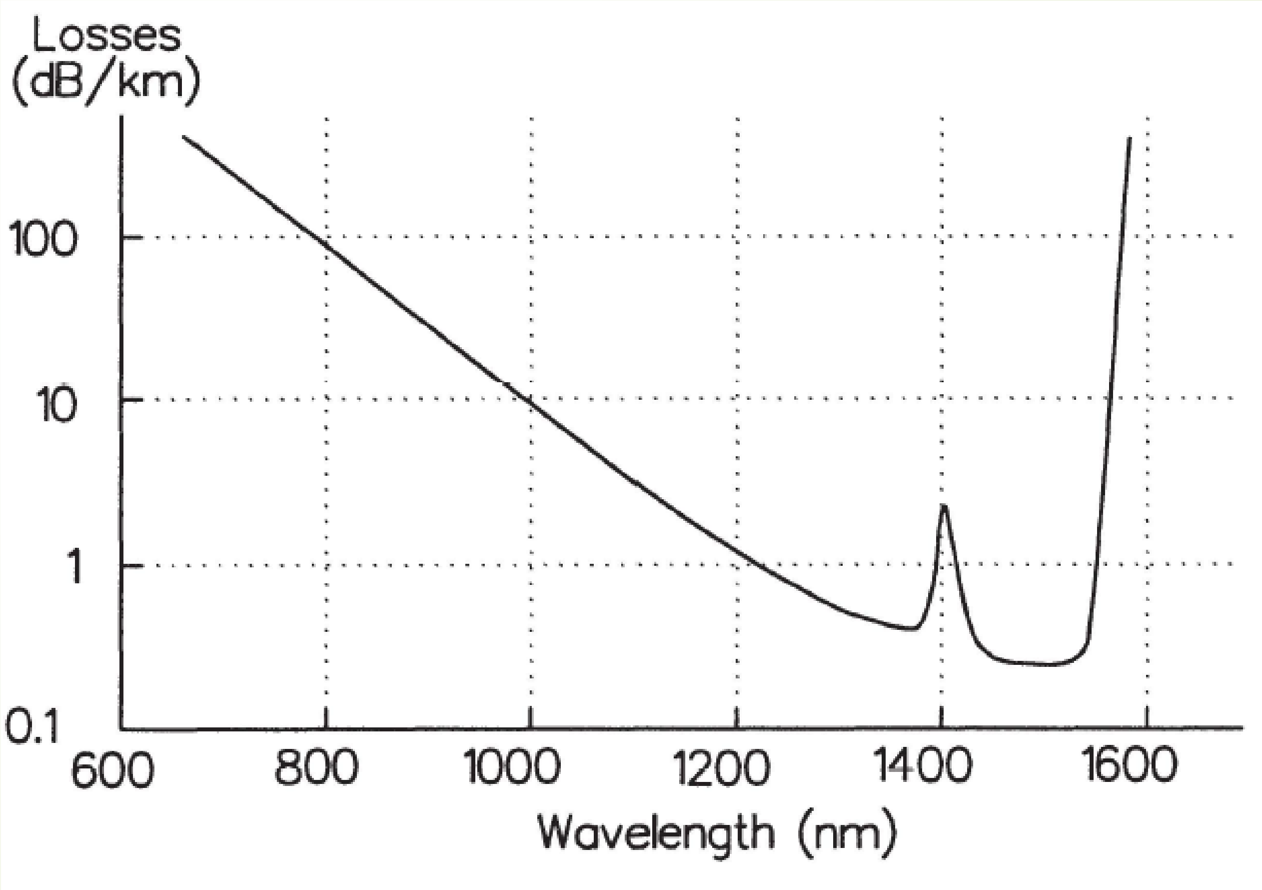
- The power in the optical signal in a fiber decreases exponentially with distance.
- $P(z)$ is the power at a position z from the origin, $P(0)$ is the power in the fiber at the origin,
- and α_p is the fiber attenuation coefficient (in units of m^{-1}, cm^{-1}, or km^{-1}).

$$P(z) = P(0)e^{-\alpha_p z}$$

$$\alpha_p = -\left(\frac{1}{z}\right)\ln\left(\frac{P(z)}{P(0)}\right)$$

$$\alpha = -\frac{10}{z[\text{km}]}\log\left(\frac{P(z)}{P(0)}\right).$$ *decibels per kilometer (dB/km).*

- the attenuation factor depends greatly on the fiber material and the manufacturing tolerances
- there is an optimum operating wavelength (1550 nm for silica fibers)



Example: An optical fiber has losses of 0.6 dB/km at 1300 nm. If 100 μW of power is injected into the fiber at the transmitter, how much will the power be at a distance of 22 km down the fiber?

$$P(\text{dBm}) = 10\log\left(\frac{P_{\text{in}}(\text{watts})}{1\times10^{-3}}\right)$$

$$\begin{aligned} P(\text{dBm}) &= 10\log\left(\frac{P_{\text{in}}(\text{watts})}{1\times10^{-3}}\right) \\ &= 10\log\left(\frac{100\times10^{-6}}{1\times10^{-3}}\right) = 10\log(10^{-1}) = -10.0\ \text{dBm} \end{aligned}$$

The output power is reduced by 0.6 dB/km times the distance of 22 km (= $0.6\times22 = 13.2$ dB). Subtracting the losses, we have

$$P_{\text{out}}(\text{dBm}) = P_{\text{in}}(\text{dBm}) - \text{losses(dB)} = -10 - 13.2 = -23.2\ \text{dBm}.$$

$$P_{\text{out}} = 10^{-23.2/10} = 4.78\times10^{-3}\ \text{mW} = 4.78\ \mu\text{W}$$

Fiber losses are due to several effects

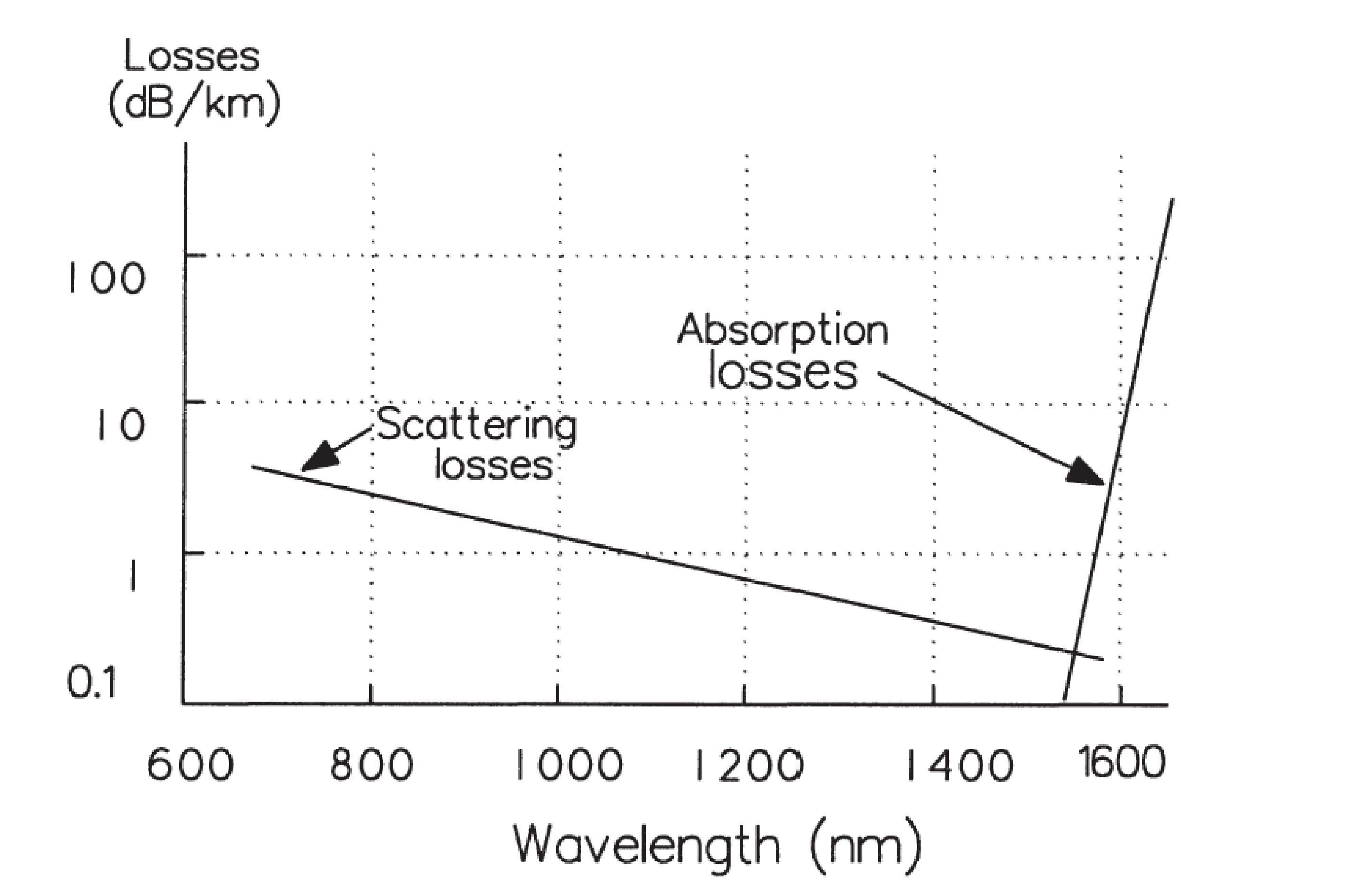
- material absorptions,
- impurity absorptions
- scattering effects
- interface inhomogeneities
- radiation from bends

Material Absorptions

- due to the molecules of the basic fiber material
- can be overcome only by changing the fiber material
- **Impurity Ions**

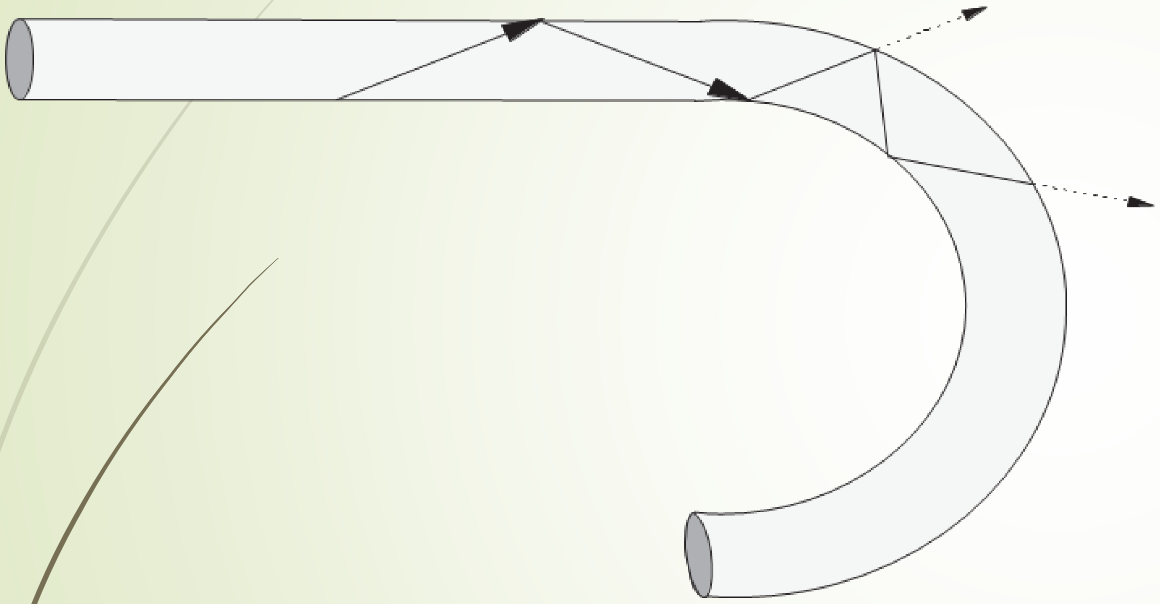
Scattering Losses

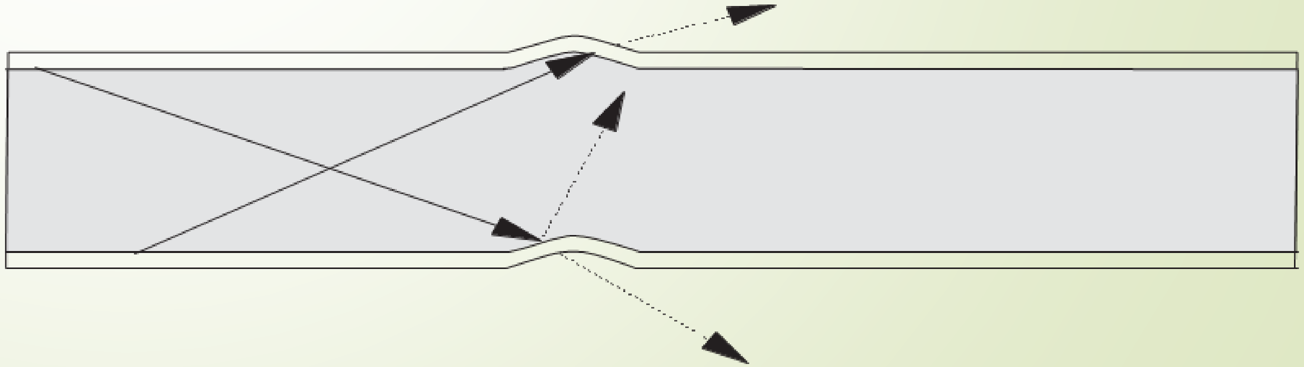
- Scattering losses occur when a wave interacts with a particle in a way that removes energy in the directional propagating wave and transfers it to other directions.
- Linear Scattering
  - ***Rayleigh scattering:*** light interacting with inhomogeneities in the medium that are much smaller than the wavelength of the light (minute changes in the refractive index of the glass at some locations)
    - This scattering strength is proportional to $1/\lambda^4$
  - ***Mie scattering:*** occurs at inhomogeneities that are comparable in size to a wavelength
    - core-cladding refractive index variations
    - impurities at the core-cladding interface
    - strains or bubbles in the fiber
    - diameter fluctuations


Losses (dB/km)
100
10
1
0.1
Absorption losses
Scattering losses
600
800
1000
1200
1400
1600
Wavelength (nm)

- Nonlinear Scattering
  - High values of electric field within the fiber
  - ***Brillouin scattering***: modulation of the light by the thermal energy in the material.
    - The scattered light is found to be frequency modulated by the thermal energy,
    - is mainly in the backward direction toward the source
  - ***Raman scattering:*** the nonlinear interaction produces a high-frequency phonon and scattered photon

Macrobending and Microbending Losses





Bend Losses: Multimode aml Single-Mode Fibers

$$\frac{P_{\text{out}}}{P_{\text{in}}} = e^{-\alpha_{\text{bends}} z}$$

$$\alpha_{\text{bends}} = c_1 e^{-c_2 r}$$

$$r_{\text{critical}} \approx \frac{3 n_2 \lambda}{4\pi (\text{NA})^3}$$

Example: (a) Calculate the critical radius of curvature for a multimode 50/125 fiber with an NA of 0.2 operating at 850 nm.

We will assume a value of $n_2 = 1.48$.

$$r_{\text{critical}} \approx \frac{3n_2\lambda}{4\pi(\text{NA})^3} \approx \frac{3(1.48)(850 \times 10^{-9})}{4\pi(0.2)^3} \approx 37.5\ \mu\text{m}$$

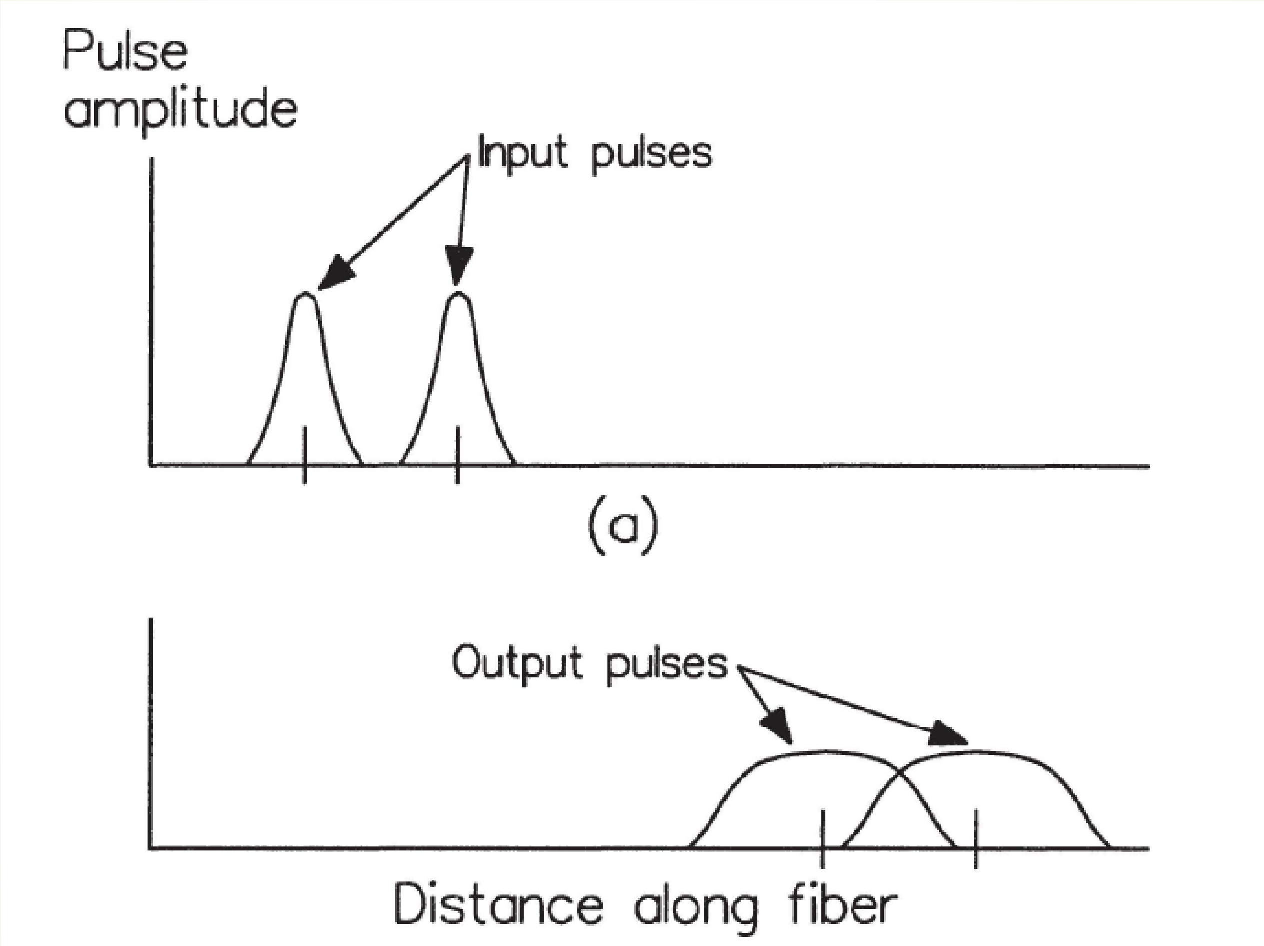
(b) For a 9/125 single-mode fiber with an NA of 0.08 operating at 1300 nm?

$$r_{\text{critical}} \approx \frac{3n_2\lambda}{4\pi\text{NA}^3} \approx \frac{3(1.48)(1300 \times 10^{-9})}{4\pi(0.08)^3} \approx 897\ \mu\text{m}$$

Dispersion

- material dispersion
- waveguide dispersion
- modal delay (dispersion)

- material dispersion and waveguide dispersion are caused by the dependence of the index of refraction of glass on wavelength and are named chromatic dispersion


Pulse amplitude
Input pulses
(a)
Output pulses
Distance along fiber

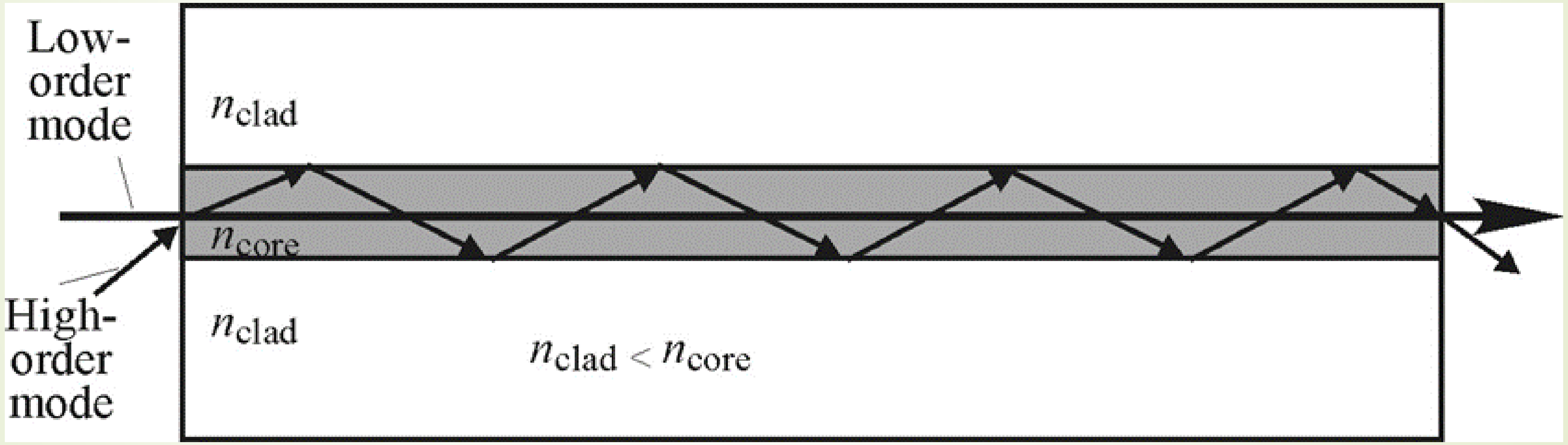
Modal Dispersion

- It is caused by the different path lengths associated with each of the modes of a fiber, as well as the differing propagation coefficients associated with each mode

Modal Dispersion I: Step-Index Fiber

$$\Delta\tau_{\text{modal}} = \frac{L(n_1 - n_2)}{c}\left(1 - \frac{\pi}{V}\right)$$

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{2\pi a}{\lambda} n_1 \sqrt{2\Delta}$$



An approximation $$\Delta\tau_{\text{modal}} \approx \frac{L\,\Delta\, n_1}{c}$$

Example: Consider a 50/125 step-index fiber with $n_1 = 1.47$ and $\Delta = 1.5\%$. Calculate the group delay (or modal dispersion) in units of ns·km^{-1} for this fiber at an operating wavelength of 850 nm.

$$\frac{\Delta\tau_{\text{modal}}}{L} \approx \frac{\Delta n_1}{c} = \frac{(0.015)(1.47)}{3 \times 10^8}$$
$$= 7.35 \times 10^{-11}\ \text{s} \cdot \text{m}^{-1} = 73.5\ \text{ns} \cdot \text{km}^{-1}$$

<u>Alternative solution:</u> We can use the more exact formula,

$$\frac{\Delta\tau_{\text{modal}}}{L} = \frac{(n_1 - n_2)}{c}\left(1 - \frac{\pi}{V}\right)$$

$$n_1 - n_2 \approx \Delta n_1 \approx (0.015)(1.47) \approx 2.21 \times 10^{-2}$$

$$V = \frac{2\pi a}{\lambda} n_1 \sqrt{2\Delta}$$
$$= \left(\frac{2\pi(25 \times 10^{-6})}{850 \times 10^{-9}}\right)(1.47)\left(\sqrt{2(0.015)}\right) \approx 47.1$$

$$\frac{\Delta\tau_{\text{modal}}}{L} \approx \frac{(n_1 - n_2)}{c}\left(1 - \frac{\pi}{V}\right) = \frac{(2.21 \times 10^{-2})}{3 \times 10^8}\left(1 - \frac{\pi}{47.1}\right) = 6.87 \times 10^{-11}\ \text{s} \cdot \text{m}^{-1} = 68.7\ \text{ns} \cdot \text{km}^{-1}.$$

- Knowing the pulse spread $\Delta\tau$, bandwidth can be found as:

$$B_{R\max} = \frac{1}{4\Delta\tau_{\text{modal}}}$$

Modal Dispersion: Graded-Index Fiber

$$\Delta\tau_{\text{modal}} \approx \begin{cases} n_1\Delta\dfrac{(g-g_{\text{opt}})L}{(g+2)c} & g \neq g_{\text{opt}} \\ \dfrac{n_1\Delta^2 L}{2c} & g = g_{\text{opt}}\,. \end{cases}$$

$$g_{\text{opt}} \approx 2 - \frac{12\Delta}{5}$$

We observe that $\Delta\tau_{\text{modal}}$ can be positive or negative depending on the size of g relative to g_{opt}. For negative $\Delta\tau_{\text{modal}}$, the interpretation is that the higher-order modes are arriving before the lower-order modes.

$$n(r) = n_1\sqrt{1 - 2\Delta\left(\frac{r}{a}\right)^g}.$$

Example: Consider a graded-index fiber with $\Delta = 2\%$ and $g_{\text{opt}} = 2.0$. If $g = 95\%$ of g_{opt}, calculate the ratio of $\Delta\tau_{\text{modal}}|_{g=g_{\text{opt}}}$ to $\Delta\tau_{\text{modal}}|_{g\neq g_{\text{opt}}}$.

Solution: We have $g = 0.95 g_{\text{opt}} = 0.95(2.0) = 1.90$, so

$$\frac{\Delta\tau_{\text{modal}}|_{g\neq g_{\text{opt}}}}{\Delta\tau_{\text{modal}}|_{g=g_{\text{opt}}}} = \frac{n_1\Delta \dfrac{g-g_{\text{opt}}}{(g+2)c} L}{\dfrac{n_1\Delta^2 L}{2c}} = \frac{2(g-g_{\text{opt}})}{\Delta(g+2)}$$

$$= \frac{(2)(1.90-2)}{(0.02)(1.90+2)} = -2.56 = -256\% .$$

Example: (a) Calculate the ratio of the modal delay per km in a 50/125 graded-index fiber with $n_1 = 1.46$, $\Delta = 1.5\%$, and $g = g_{opt} = 2$ to the modal delay in a step-index fiber of the same size with the same n_1 and Δ.

Solution: The time delays are given by

$$\frac{n_1 \Delta\tau(\mathrm{GI})|_{g=g_{opt}}}{L} \approx \frac{\Delta^2}{2c}$$

$$\frac{\Delta\tau(\mathrm{SI})}{L} \approx \frac{n_1 \Delta}{c}.$$

Taking the ratio,

$$\frac{\Delta\tau(\mathrm{GI})|_{g=g_{opt}}}{\Delta\tau(\mathrm{SI})} = \frac{\frac{n_1\Delta^2}{2c}}{\frac{n_1 \Delta}{c}} = \frac{\Delta}{2} = \frac{0.015}{2} = 0.00750.$$

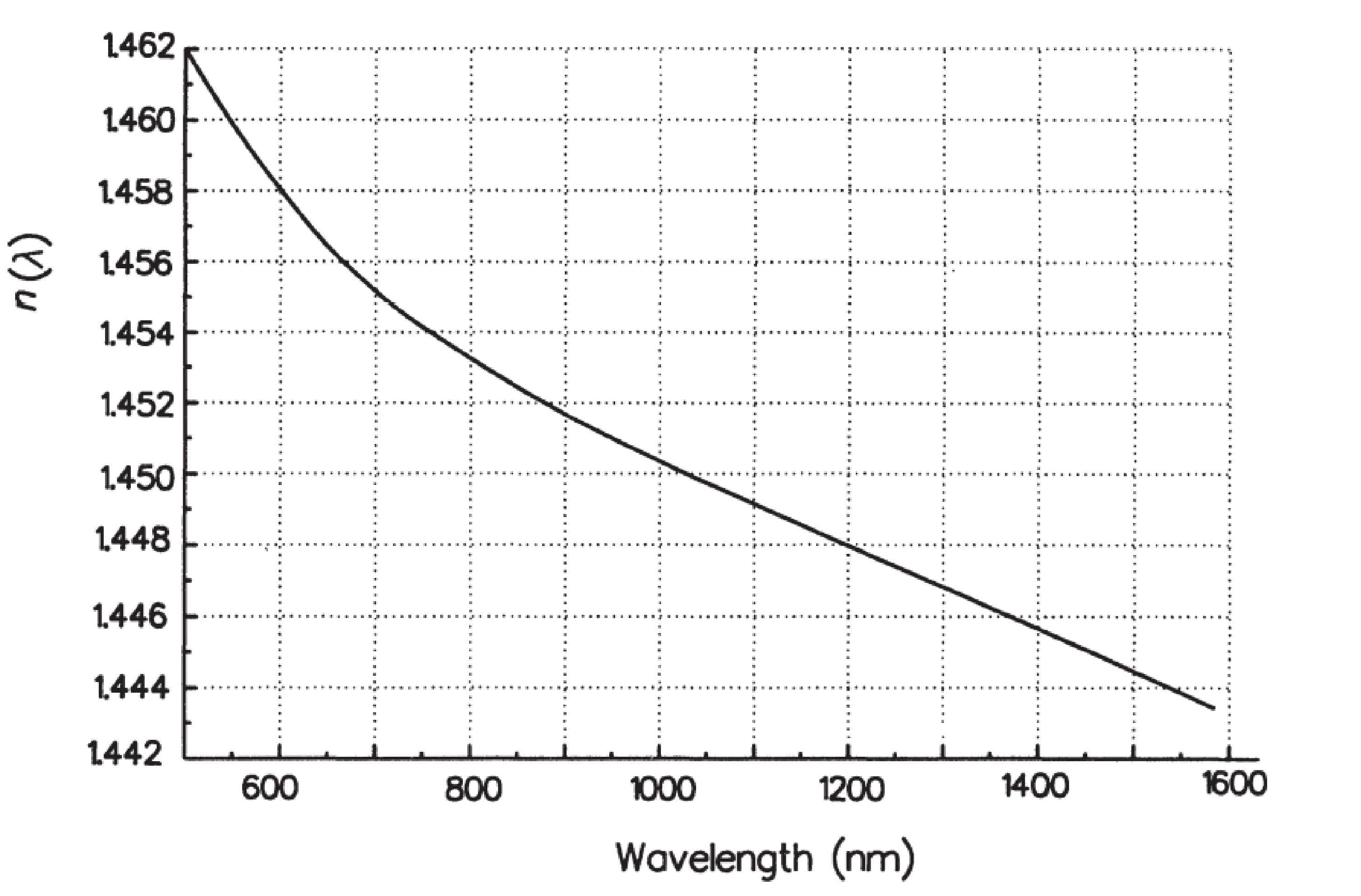
(b) Consider the same question if the graded-index fiber is not optimized. Let $g = 2.1$ and $g_{\text{opt}} = 2.0$.

$$\frac{\Delta\tau(\text{GI})|_{g\neq g_{\text{opt}}}}{\Delta\tau(\text{SI})} = \frac{\dfrac{n_1\Delta(g-g_{\text{opt}})}{(g+2)c}}{\dfrac{n_1\Delta}{c}} = \frac{g-g_{\text{opt}}}{(g+2)}$$

$$= \frac{2.1-2.0}{4.1} = 0.0244\,.$$

Material Dispersion

- Caused by the index of refraction as it depends on wavelength



Example: Derive the expression for the material dispersion in a fiber.

Solution: The arrival time τ of light after traversing a length L of fiber is

$$\tau = L/v_g \, ,$$

where v_g is the group velocity of the fiber, given by

$$v_g = \frac{1}{\frac{d\beta}{d\omega}} \, .$$

We have, then,

$$\tau = L\frac{d\beta}{d\omega} = L\,\frac{d\beta}{d\lambda}\,\frac{d\lambda}{d\omega} \, .$$

Since $\lambda = c/\nu = 2\pi c/\omega$, we find

$$\frac{d\lambda}{d\omega} = -\frac{2\pi c}{\omega^2} = -\frac{1}{\omega}\frac{2\pi c}{\omega} = -\frac{\lambda}{\omega} \, .$$

Substituting Eq. 3.28 into Eq. 3.27, we obtain

$$\tau = L\frac{d\beta}{d\lambda}\left(-\frac{\lambda}{\omega}\right) = -\frac{L\lambda}{\omega}\frac{d\beta}{d\lambda} = -\frac{L\lambda^2}{2\pi c}\frac{d\beta}{d\lambda} \, .$$

We know that $\beta = 2\pi n(\lambda)/\lambda$, so

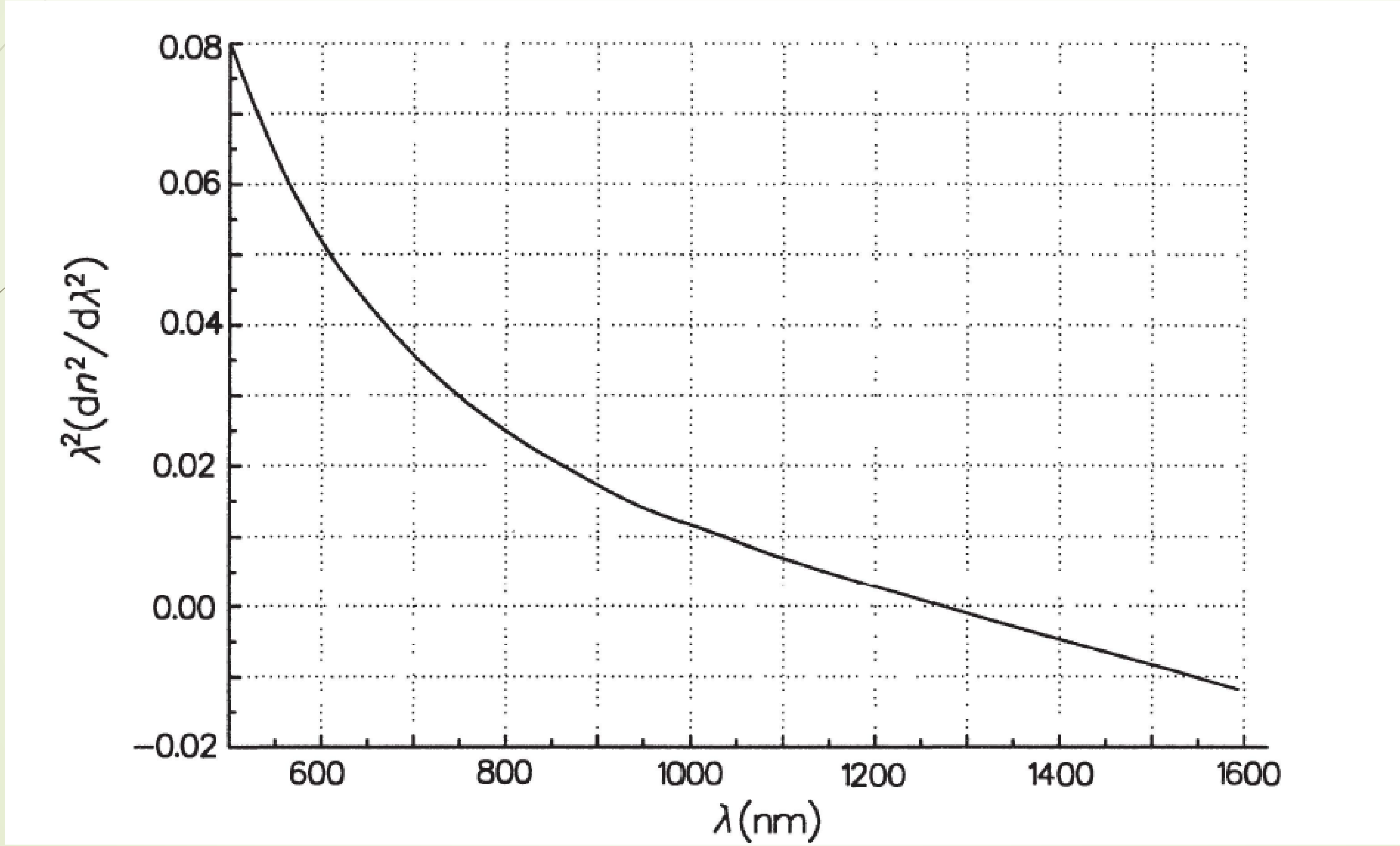
$$\begin{aligned} \tau &= -\frac{L\lambda^2}{2\pi c}\frac{d\beta}{d\lambda} \\ &= -\frac{L\lambda^2}{2\pi c}\left[-\frac{2\pi n}{\lambda^2} + \frac{2\pi n'}{\lambda}\right] \\ &= -\frac{L}{c}\left[-n + \lambda n'\right] = +\frac{L}{c}\left[n(\lambda) - \lambda\frac{dn(\lambda)}{d\lambda}\right] . \end{aligned}$$

The pulse spread $\Delta\tau$ due to a source linewidth of $\Delta\lambda$ is

$$\frac{\Delta\tau}{\Delta\lambda} = \frac{d\tau}{d\lambda} = \frac{L}{c}\left[\frac{dn(\lambda)}{d\lambda} - \lambda\frac{d^2n}{d\lambda^2} - \frac{dn}{d\lambda}\right] = -\frac{L\lambda}{c}\frac{d^2n}{d\lambda^2} .$$

Multiplying by $\Delta\lambda$, we find the desired expression for the material dispersion,

$$\Delta\tau = -\frac{L\lambda\,\Delta\lambda}{c}\frac{d^2n}{d\lambda^2} = -\frac{L}{c}\frac{\Delta\lambda}{\lambda}\left(\lambda^2\frac{d^2n}{d\lambda^2}\right) .$$



0.08
0.06
0.04
0.02
0.00
−0.02
$\lambda^2(dn^2/d\lambda^2)$
600
800
1000
1200
1400
1600
λ (nm)

Example: Consider the material dispersion in a 62.5/125 fiber with $n_1 = 1.48$ and $\Delta = 1.5\%$.

(a) Calculate the material dispersion in normalized units of ps·km^{-1}·nm^{-1} at 850 nm.

Solution: The pulse spreading is

$$\Delta\tau_{\text{mat}} = -\frac{L}{c}\frac{\Delta\lambda}{\lambda}\left(\lambda^2\frac{d^2n_1}{d\lambda^2}\right).$$

The normalized delay is

$$\frac{\Delta\tau_{\text{mat}}}{L\,\Delta\lambda} = -\frac{1}{c\lambda}\left(\lambda^2\frac{d^2n}{d\lambda^2}\right).$$

From Fig. 3.8, we see that $\lambda^2 d^2n_1/d\lambda^2$ is approximately 0.022 at $\lambda = 850$ nm; hence,

$$\begin{aligned}\frac{\Delta\tau_{\text{mat}}}{L\,\Delta\lambda} &= -\frac{1}{c\lambda}\left(\lambda^2\frac{d^2n}{d\lambda^2}\right) = -\frac{1}{(3.0\times10^8)(850\times10^{-9})}(0.022) \\ &= -8.63\times10^{-5}\ \text{s}\cdot\text{m}^{-1}\cdot\text{m}^{-1} = -86.3\ \text{ps}\cdot\text{km}^{-1}\cdot\text{nm}^{-1}.\end{aligned}$$

(b) ... at 1500 nm?

Solution: From Fig. 3.8, we estimate that $\lambda^2 d^2 n_1/d\lambda^2 \approx -0.007$, so

$$\frac{\Delta\tau_{\text{mat}}}{L\,\Delta\lambda} = -\frac{1}{c\lambda}\left(\lambda^2\frac{d^2n}{d\lambda^2}\right) = -\frac{1}{(3.0\times10^8)(1500\times10^{-9})}(-0.007)$$

$$= +1.55\times10^{-5}\ \text{s}\cdot\text{m}^{-1}\cdot\text{m}^{-1} = +15.5\ \text{ps}\cdot\text{km}^{-1}\cdot\text{nm}^{-1}\,.$$

Waveguide Dispersion

- For the low material-dispersion region near 1.27 μm, *waveguide dispersion* becomes important.
- is negligible in multimode fibers and in single-mode fibers operated at wavelengths below 1 μm,
- it is not negligible for single-mode fibers operated in the vicinity of 1.27 μm.
- results from the propagation constant of a mode (and, hence, its velocity) being a function of a/λ.

$$\tau_{wg} = \frac{L}{c}\frac{d\beta}{dk} .$$

β is the mode's propagation coefficient and $k = 2\pi/\lambda$

We again define the normalized propagation constant b as

$$b = \frac{(\beta^2/k^2) - n_2^2}{n_1^2 - n_2^2} .$$

An approximation for b is

$$b \approx \frac{(\beta/k) - n_2}{n_1 - n_2} ,$$

thereby giving

$$\beta \approx n_2 k\,(b\Delta + 1) .$$

Here b is a function of V (and of k)

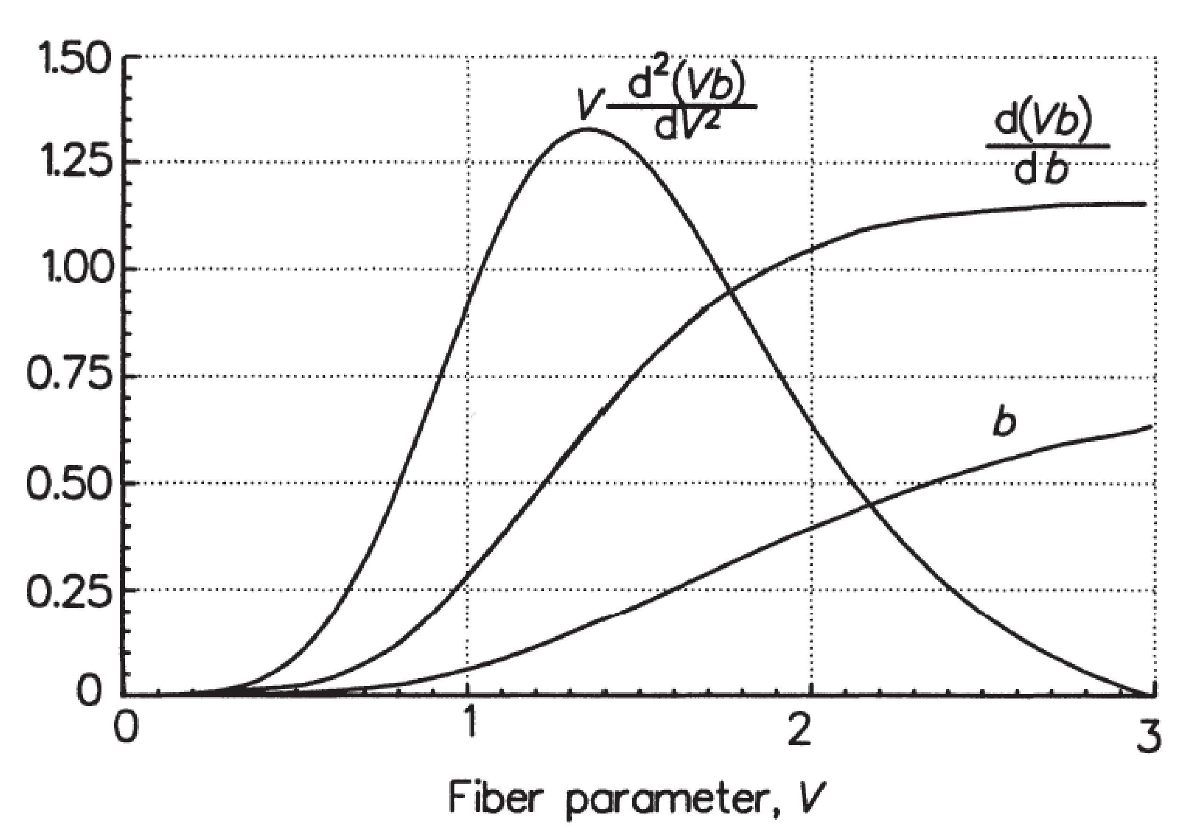
$$\tau_{wg} \approx \frac{L}{c}\left(n_2 + n_2\Delta\frac{d(kb)}{dk}\right)$$

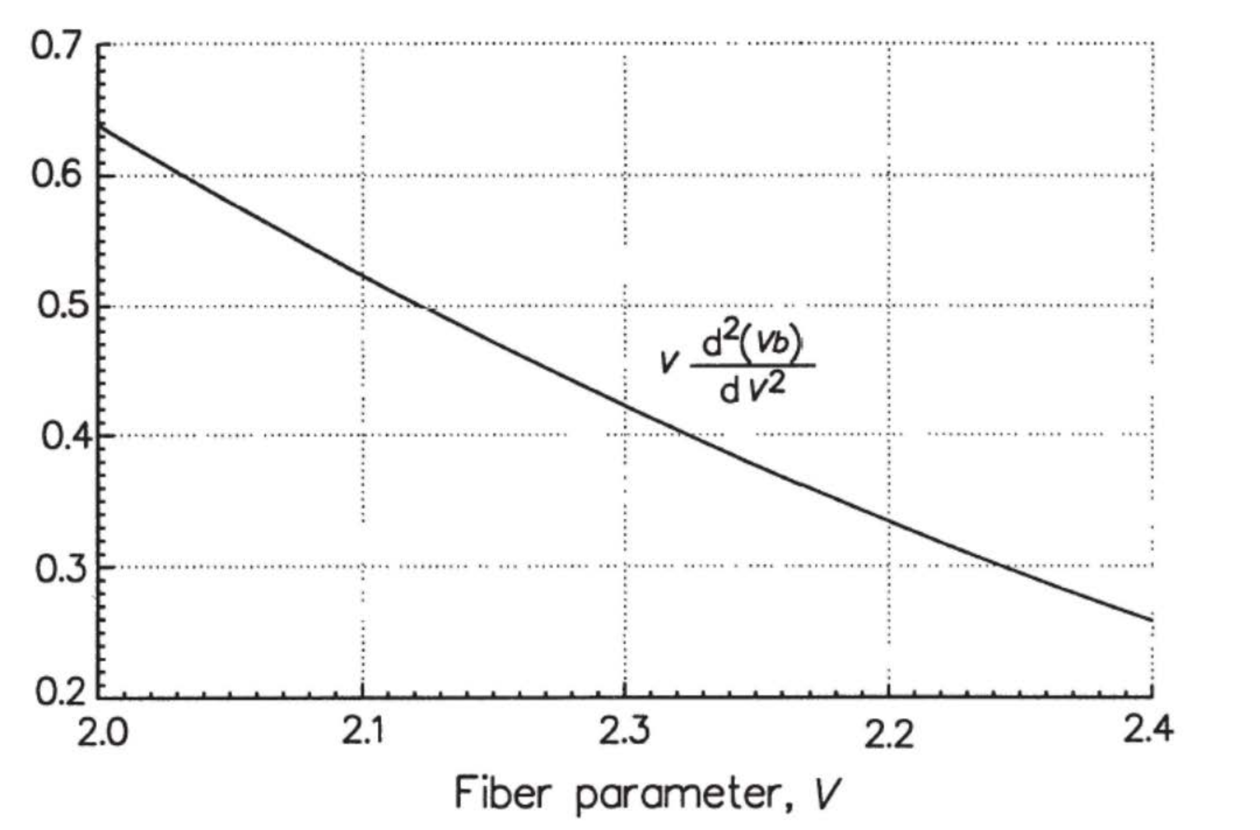
$$V \approx kan_2\sqrt{2\Delta},$$

$$\tau_{wg} \approx \frac{L}{c}\left(n_2 + n_2\Delta\frac{d(Vb)}{dV}\right)$$

$$\tau_{wg}(\lambda) \approx \frac{n_2\Delta\, L}{c}\frac{d(Vb)}{dV}$$

Plot of b, $d(Vb)/dV$, and $V\,d^2(Vb)/dV^2$ vs. V for the lowest-order fiber mode.





Since $V = 2\pi a n_1 \sqrt{2\Delta}$, we can show that $dV/d\lambda = -V/\lambda$.

normalized propagation constant, b,

$$b(V) = 1 - \frac{u^2}{V^2} = \frac{\left(\frac{\beta^2}{k^2}\right) - n_2^2}{n_1^2 - n_2^2}$$

$$\beta = k\sqrt{n_2^2 + (n_1^2 - n_2^2)b}$$

$$b = \frac{\frac{\beta^2}{k^2} - n_2}{n_1^2 - n_2^2} \approx \frac{\frac{\beta}{k} - n_2}{n_1 - n_2} .$$

$$b(V) = 1 - \left(\frac{(1+\sqrt{2})^2}{\sqrt{1+(4+V^4)}}\right)$$

$$\Delta\tau_{wg} = -\frac{V}{\lambda}\Delta\lambda\frac{d\tau_{wg}}{dV} \approx -\frac{n_2 L\Delta}{c}\frac{\Delta\lambda}{\lambda}\left(V\frac{d^2(Vb)}{dV^2}\right)$$

$$b(V) = 1 - \left(\frac{(1+\sqrt{2})^2}{\sqrt{1+(4+V^4)}}\right)$$

$$V = 2\pi a n_1\sqrt{2\Delta},$$

Example: Calculate the waveguide dispersion in units of ps·km^{-1}·nm^{-1} for a 9/125 single-mode fiber with $n_1 = 1.48$ and $\Delta = 0.22\%$ operating at 1300 nm.

Solution: We begin by calculating V from Eq. 2.9 on page 15,

$$V = \frac{2\pi a}{\lambda} n_1 \sqrt{2\Delta} = \frac{2\pi(4.5 \times 10^{-6})}{1300 \times 10^{-9}}(1.48)\sqrt{2(0.0022)} = 2.14\,. \tag{3.52}$$

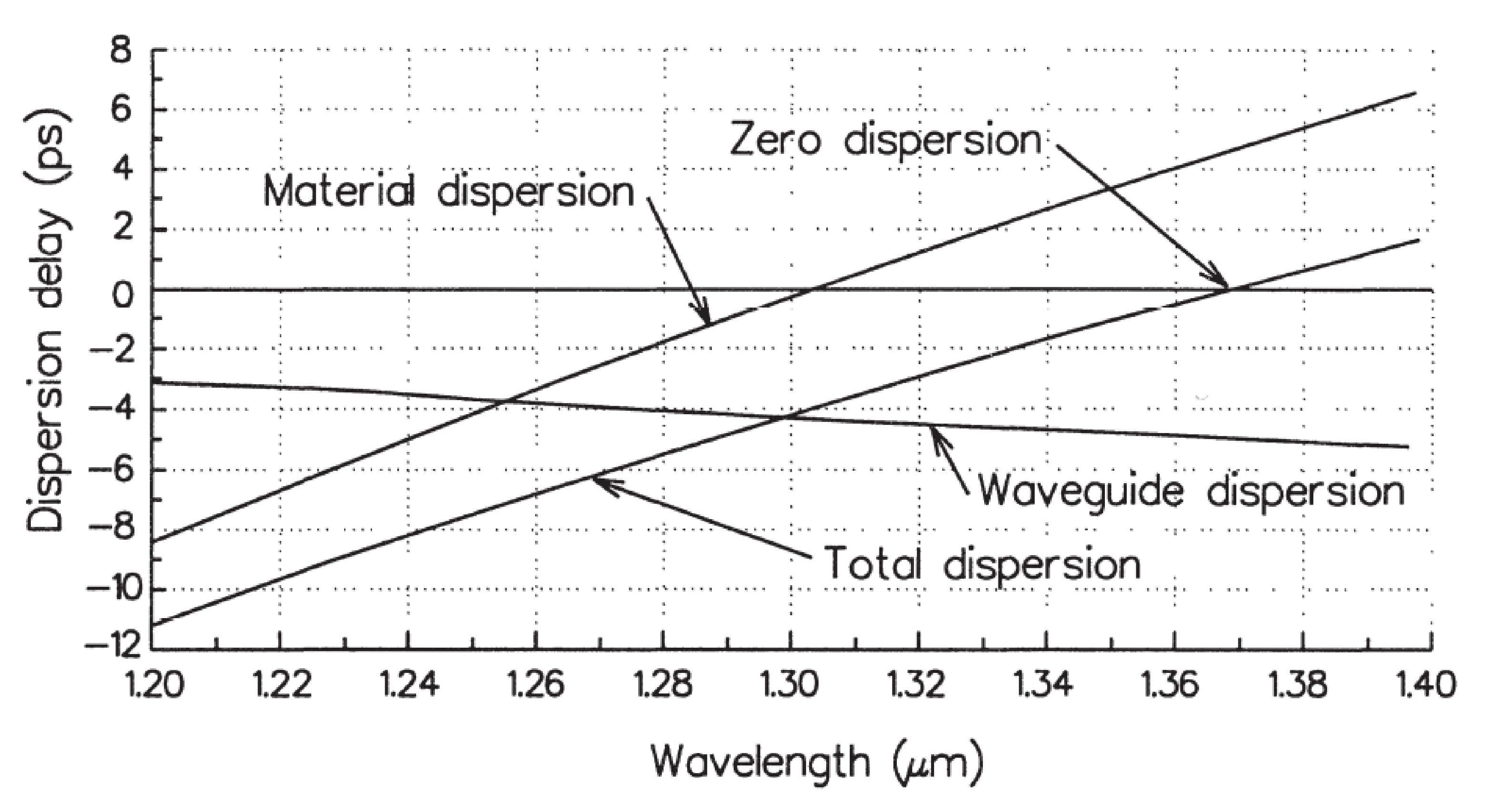
(We note that V falls within the expected range of $2.0 < V < 2.405$ for single-mode fiber.) From Fig. 3.10, we find $V\, d^2(Vb)/dV^2 \approx 0.480$ at $V = 2.14$.

We also have $n_2 = n_1(1 - \Delta) = 1.48(1 - 0.0022) = 1.477$, so

$$\frac{\Delta\tau_{wg}}{L\,\Delta\lambda} = -\left(\frac{n_2\Delta}{c}\right)\left(\frac{1}{\lambda}\right)\left(V\frac{d^2(Vb)}{dV^2}\right) \tag{3.53}$$

$$
\begin{aligned}
&= -\left(\frac{(1.477)(0.0022)}{3 \times 10^{8}}\right)\left(\frac{1}{1300 \times 10^{-9}}\right)(0.48) \\
&= -4.00 \times 10^{-6} = -4.00\ \text{ps} \cdot \text{km}^{-1} \cdot \text{nm}^{-1}\,.
\end{aligned}
$$

Total Dispersion: Single-Mode Fiber



- To minimize the total dispersion of a single-mode fiber, it is necessary to operate at a wavelength longer than 1.27 μm to allow the small positive material dispersion to cancel the small negative waveguide dispersion.
- This zero dispersion point occurs near 1300 nm, a wavelength that, fortunately, has a fairly low attenuation (although not as low as the attenuation minimum at 1550 nm)
- waveguide dispersion has been found to be sensitive to the doping levels as well as the values of Δ and a.
- For various combinations of Δ and a, and for triangular and other profiles zero dispersion at wavelengths between 1300 and 1700 nm are possible

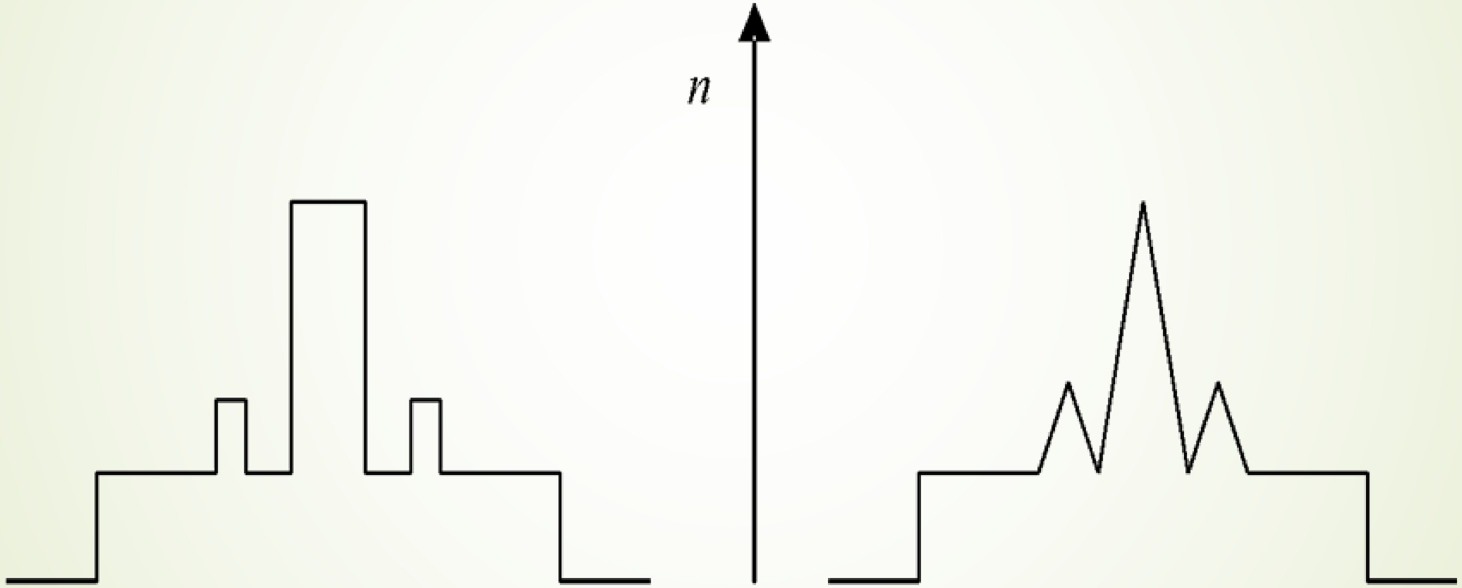
Dispersion-Adjusted Single-Mode Fibers

- lowest losses occur at a 1500 nm wavelength
- lowest total dispersion occurs (in a step-index single-mode fiber) at 1300 nm
- The two features can be combined using dispersion shifting or move the zero-dispersion wavelength to 1550 nm (lowest loss wavelength)

Dispersion-Shifted Fibers

- material dispersion of silica can be adjusted in small amounts by doping the core
- waveguide dispersion depends on the fiber-core radius, Δ

Dispersion-Shifted Multi-Index Fibers
More complicated



W-profile fibers:

(a) step-index, (b) triangular profile

Total Dispersion

- $\Delta t_{total} = \left(\Delta t_{modal}^2 + \Delta t_{chromatic}^2\right)^{\frac{1}{2}}$
- $BW\ (Hz) \approx \frac{0.35}{\Delta t_{total}}$

Example

A 2-km-length multimode fiber has a modal dispersion of $1\, ns/km$ and a chromatic dispersion of $100 ps/km.nm$. It is used with an LED of linewidth $40\, nm$. (a) What is the total dispersion? (b) Calculate the bandwidth (BW) of the fiber.

$$\Delta t_{modal} = 2\, km \times 1\, ns/km = 2\, ns$$

$$\Delta t_{chromatic} = (2\, km) \times (100\, ps/km\, nm) \times (40\, nm)$$

$$= 8000\, ps = 8\, ns$$

$$\Delta t_{total} = ([2\, ns]^2 + [8\, ns]^2)^{\frac{1}{2}} = 8.25\, ns$$

$$BW = \frac{0.35}{\Delta t_{total}} = \frac{0.35}{8.25\ ns} = 42.42\, MHz$$

Expressed in terms of the product $(BW.km)$, we get $(BW.km) = (42.4\, MHz)(\,2\, km) \cong 85\, MHz.km$.

Example

A 50-km single-mode fiber has a material dispersion of $10ps/km.nm$ and a waveguide dispersion of $-5ps/km.nm$. It is used with a laser source of linewidth 0.1 nm. (a) What is $\Delta t_{chromatic}$? (b) What is Δt_{total}? (c) Calculate the bandwidth (BW) of the fiber.

$$\Delta t_{chromatic} = 10ps/km.nm - 5ps/km.nm = 5ps/km.nm$$

For 50 km of fiber at a line width of 0.1 nm, Δt_{total} is

$$\Delta t_{total} = (50\ km) \times (5ps/km.nm) \times (0.1\ nm) = 25\ ps$$

(b) $BW = 0.35/\Delta t_{total} = 0.35/25\ ps = 14\ GHz$

Expressed in terms of the product (BW. km), we get

$$(BW.km) = (14\ GHz)(50\ km) = 700\ GHz.km$$

In short, the fiber in this example could be operated at a data rate as high as 700 GHz over a one-kilometer distance.